

# FERROELASTICITY OF MATERIALS WITH A MECHANICAL SHAPE MEMORY

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It is extremely difficult to take into account the variety of aspects of the behavior of solids when they are deformed. Hence, when designing constructions, simplifying models are introduced which take into account only the most important properties of the materials in each specific case (creep, aftereffect, plasticity, etc.), and the corresponding phenomenological theories are employed [1]. In this paper we attempt to construct the fundamentals of a theory which describes the phenomenon of ferroelasticity due to the behavior of thermoelastic martensite, first discovered in [2]. The apparatus of this theory will be necessary when designing self-recovering constructions which can be manufactured from materials with a mechanical shape memory.

1. It has recently been found that a large number of alloys (NiTi, CuAlNi, CuZn, CuZnSi, AlCuZn, FeNi, FeMn, NiAl, etc.) and certain pure metals (Co, Ti, Zr) undergo a thermoelastic martensitic transition, which is accompanied by a number of unique properties: superelasticity, a shape memory effect, spontaneous directional deformation when cooled, etc. [3-7].

The most interesting effect (from the point of view of its technological use) is the shape memory effect, which is as follows. If the material, which is in the high-temperature modification, is given a certain geometrical shape, and then, when it is cooled through the martensite-transition temperature it is plastically deformed, when it is then heated above the transition point the material reverts to its initial specified shape. The value of the completely reversible inelastic deformation is not the same for different materials and is 6-16% [3, 5].

Figure 1 shows a schematic family of graphs illustrating the most important features of the mechanical properties of alloys with thermoelastic martensite at different temperatures. Curve 1 represents the usual plastic flow, curves 2 and 3 represent the deformation as a function of the stress when the superelasticity effect is present, and curve 4 illustrates the effect which has come to be called ferroelasticity and is directly responsible for the mechanical shape memory [8]. The temperatures  $T_{AM}$  and  $T_{MA}$  correspond to the forward and reverse martensitic transitions. All the graphs are symmetrical about the origin of coordinates. Note that the term "ferroelasticity" has been introduced because of the resemblance between the shape of the graphs of the magnetization of ferromagnetic materials as a function of the external magnetic field, and the deformation as a function of the stress for materials with shape memory.

Investigations of the microstructure of martensite show that it consists of different kinds of structural formations (twin plates, packing defects, superdislocations, etc. [9]), which have been called domains. The domains possess a residual deformation  $\alpha_{ij}$  compared with the high-temperature phase. Its intensity  $\alpha = \sqrt{\alpha_{ij}\alpha_{ij}}$  is the same for all domains and is determined by the crystal-geometrical features of the phase transition. The orientation, which can be specified by the unit vector in deformation space with components  $k_{ij} = \alpha_{ij}\alpha^{-1}$ , can be both random and ordered. Averaging over the volume of the specimen therefore leads, respectively, to zero or different from zero residual macrodeformations

$$e_{ij}^n = \frac{1}{V} \int_V \alpha_{ij} dV = \alpha \langle k_{ij} \rangle.$$

They obviously reach a maximum value when there is complete ordering of the orientation of the vector with components  $k_{ij}$  in a certain direction. Hence, the motion of the domain walls in actual space, which leads to storage of inelastic deformations, corresponds to reorientation of the unit vectors in deformation space.

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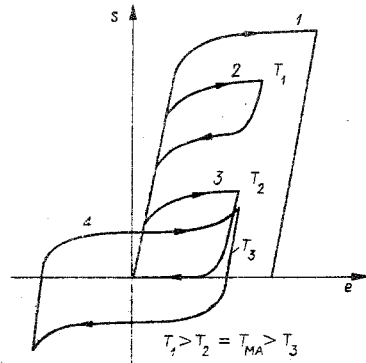


Fig. 1

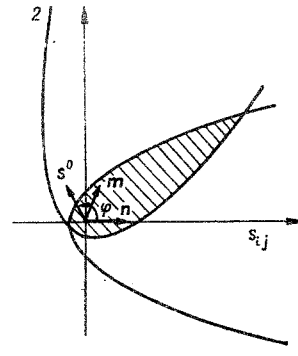


Fig. 2

Note that the hydrostatic pressure does not have any appreciable effect on the thermoelastic martensitic transition, but the change in the specific volume of the phases through the transition is negligibly small for the majority of materials [5], so that all the tensors considered here coincide with their deviators.

2. We will consider the deformation below the temperature  $T_{MA}$ . We will write the thermodynamic Gibbs potential of unit volume of the isotropic martensitic phase having maximum inelastic deformations characterized by the unit vector with components  $n_{ij}$ :

$$-G_n = \frac{1}{2} \mu s_{ij} s_{ij} + \alpha n_{ij} s_{ij} + g(T), \quad (2.1)$$

where  $\mu$  is the inverse of twice the shear modulus,  $s_{ij}$  is the stress tensor, and  $g(T)$  is the chemical component of the potential, which depends on temperature. It is obvious that it is the same for martensite of any orientation. The deformation tensor, as is well known, can be expressed in terms of the derivative of the Gibbs potential

$$e_{ij} = -\frac{\partial G_n}{\partial s_{ij}} = \mu s_{ij} + \alpha n_{ij} \equiv e_{ij}^v + e_{ij}^n. \quad (2.2)$$

Equations (2.1) and (2.2) have been written on the assumption that the martensite obeys Hooke's law always, including the instants of reorientation, when a change in the inelastic components of the deformation occurs. Suppose that reorientation of the martensite into the deformed state, characterized by a vector with components  $m_{ij}$ , occurs due to applied stresses. The thermodynamic potential then has the form

$$-G_m = (1/2) \mu s_{ij} s_{ij} + \alpha m_{ij} s_{ij} + g(T),$$

while the deformation tensor has the form

$$\mu s_{ij} + \alpha m_{ij}.$$

In a certain intermediate state in the volume  $V_m$  the domains will have a new orientation  $m_{ij}$ , and in the volume  $V_n = V - V_m$  as before, it is determined by the vector with components  $n_{ij}$ . Here  $V$  is the volume of the whole specimen. The macroscopic inelastic deformations, determined by averaging over the volume  $V$ , have the form

$$e_{ij}^n = \langle \alpha_{ij} \rangle = \alpha \left\{ \frac{V_m}{V} m_{ij} + \left( 1 - \frac{V_m}{V} \right) n_{ij} \right\} = \alpha n_{ij} + \lambda \alpha \{ m_{ij} - n_{ij} \}, \quad (2.3)$$

where  $\lambda = V_m V^{-1}$  is the concentration of newly formed martensite.

On the other hand, since the motion of the interdomain walls has an energetically active character, the condition for a transition from the state  $n_{ij}$  to the state  $m_{ij}$  can be written in the form

$$f(s) \equiv G_n - G_m = \alpha (m_{ij} - n_{ij}) s_{ij} = K(T), \quad (2.4)$$

where  $K(T)$  is the activation energy. Comparing Eq. (2.3) written in terms of increments and Eq. (2.4) we obtain

$$d e_{ij}^n = d \lambda (m_{ij} - n_{ij}) \alpha = d \lambda \frac{\partial f}{\partial s_{ij}}. \quad (2.5)$$

Hence, the increments in the inelastic components of the deformation tensor during reorientation of thermoelastic martensite obey the associated flow law (2.5) with the flow condition (2.4) [10]. The coefficient of proportionality  $d\lambda$  has the meaning of the increment of the concentration of the newly formed phase.

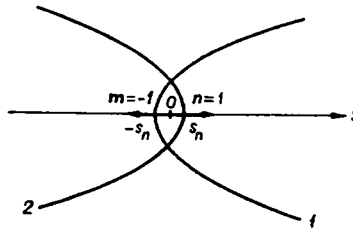


Fig. 3

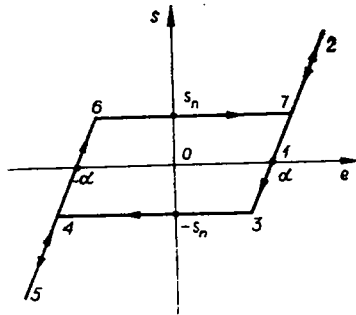


Fig. 4

Equation (2.4) defines a family of surfaces in stress space (we will call them reorientation surfaces), which depend on the parameter  $m_{ij}$ . In practice, that martensitic orientation is realized (and, consequently, that surface is fixed) for which condition (2.4) is satisfied for the minimum stress intensity  $s = \sqrt{s_{ij}s_{ij}}$ , which corresponds to the maximum work of the stresses on the inelastic deformations [4]. It can be shown that then  $m_{ij} = s_{ij}^0 \equiv s_{ij}s^{-1}$  and it follows from Eqs. (2.4) and (2.5) that

$$f_n(s) = (s - n_{ij}s_{ij})\alpha = K(T); \quad (2.6)$$

$$de_{ij}^n = d\lambda (s_{ij}s^{-1} - n_{ij}) \alpha. \quad (2.7)$$

Surface (2.6) defines the region, the stress changes in which cause only elastic deformations. Figure 2 shows a polar diagram of the dependence of  $s$  on the angle  $\varphi$  between the directions  $n_{ij}$  and  $s_{ij}^0$  in the case of a two-component stress state (curve 1). Since the components of the vector  $n_{ij}$  emerge as parameters of the history of the inelastic deformation, martensite having some other orientation  $m_{ij}$  will possess the following reorientation condition:

$$f_m(s) = (s - m_{ij}s_{ij})\alpha = K(T) \quad (2.8)$$

(see Fig. 2, curve 2). The presence of a complex structure consisting of domains with both orientations implies the simultaneous existence of two surfaces. Consequently, elastic deformation of the specimen as a whole, not touching on its structure, is possible if the stresses vary inside the hatched region.

In the case of complex loading relations (2.7) in general become nonintegrable and final relations of the form (2.3) cannot be constructed using them.

3. As an example, consider a simple alternating deformation. In particular, this can be a uniaxial extension-compression, an alternating twist, etc. Suppose that at the initial instant of time the specimen consists of domains with an orientation specified in a certain way in the chosen system of coordinates by a directional vector with a single nonzero component  $n=1$  (Fig. 3, curve 1). Its residual deformation is equal to  $e^n = \alpha$  (Fig. 4, curve 1). In the chosen system of coordinates we obtain from Eq. (2.6)

$$f_+(s) = s(1 - s^0)\alpha = K(T), \quad (3.1)$$

where  $s^0 = \pm 1$ . If the deformation occurs in the positive direction  $s^0=1$ , then condition (3.1) will not be satisfied for finite values of the stress and, consequently, in this case an elastic deformed state is realized (Sec. 1-2 in Fig. 4).

Loading in the opposite direction  $s^0=-1$  will be elastic up to a value of the stress  $s_n = K(T)/2\alpha$ , which follows from Eq. (3.1). The reorientation processes which then occur lead to the appearance of domains with a direction vector  $m=-1$  and storage by the specimen of the residual deformation  $e^n = -\alpha$ , and to replacement of the flow surface. We obtain from Eq. (2.8)

$$f_-(s) = s(1 + s^0) \alpha = K(T)$$

(Fig. 3, curve 2). It can be seen from this equation that a further increase in the stresses in the previous direction  $s^0 = -1$  will be accompanied only by elastic deformation (4-5, Fig. 4). Repeated deformation in the positive direction has an elastic region bounded by the value of the surface  $s_n = K(T)/2\alpha$ , which follows from Eq. (3.2) (point 6 in Fig. 4). Subsequent reorientation closes the cycle.

According to this theory further isothermal cycling does not lead to any change in the parameters of the hysteresis loop, which agrees with experiment [8]. Their temperature dependence manifests itself in terms of the activation energy  $K(T)$  of the reorientation mechanism. Disappearance of the loop when  $T = T_{MA}$  denotes complete return of inelastic deformation, acquired in the martensitic state [3].

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